ON THE NATURE OF QUANTUM WAVES

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Synopsis

Technology majors, when introduced to quantum concepts, invariably encounter difficulties in the understanding of matter waves. Further, de Broglie waves are usually considered to be different from the waves inherent in Schrodinger's equation. The purpose of this paper is to clarify the nature of quantum waves and to pursue the implications of wave-particle duality.

Introduction

Although the particle properties of waves were discovered in 1905, two decades were to pass before the converse was speculated. Then in 1924 de Broglie[1] employed Planck's law, the mass/energy equation, and an intuitive faith in the symmetry of nature to postulate that a particle of relativistic mass (m) and moving with a velocity (v) has an associated wave of wavelength.

$$\lambda = \frac{h}{mv}$$

where h is Planck's constant. Known as the de Broglie

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hypothesis, this notion quickly gained widespread acceptance, although experimental verification of these matter waves was not available for a few years.[2]

Schrodinger[3] accepted wave/particle duality axiomatically and developed a wave equation for atomic particles. The Schrodinger equation is central to quantum mechanics in a role somewhat analogous to Newton's second law of motion in classical dynamics. In the so-called time-independent form we can write Schrodinger's equation:

$$\nabla^2\psi + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0$$

The quantity ψ is called the wave function. It is in general a complex variable, and we require to solve Schrodinger's equation in order to obtain the wave function. Here, E is the total energy of the particle and V its potential energy. The symbol ∇^2 is an operator called the Laplacian and denotes second partial differential along the three co-ordinate axes.

Because of the dependence upon boundary conditions, we cannot give a general solution to Schrodinger's equation, but most are in trigonometric, exponential or hyperbolic form. The concept of the wave function is a prerequisite to the appreciation of matter waves; in fact, even Schrodinger did not fully understand this quantity until it was correctly interpreted by Born[4] in 1926.

Physical Significance of Matter Waves

In a light wave, the electro-magnetic field varies in space/time, in an acoustic wave the pressure varies in space/time; what variations constitute matter waves? In fact, it is the wave function which is varying. Let us now explore what this quantity is.

It should be understood at the outset that the wave function has no direct physical significance; it is a mathematical device which enables us to perform calculations in quantum mechanics. We can, however, assert that the value of the wave function associated with a moving particle at a particular point in space and time is related to the likelihood of finding the particle there at that time. The probability that an object is somewhere at a given time lies between 0 and 1, corresponding to the certainty of absence and the certainty of presence, respectively, But wave amplitudes can be positive and negative, and a negative probability is meaningless. Hence, we see that the wave function is not an observable quantity. However, this objection does not apply to the square of the modulus; physically this can be, for example, the density of particles.

The wave function, being complex, can be written in the Cartesian form

$$\psi = a + ib \qquad (i^2 = -1)$$

The first term represents a real and the second term an imaginary number. Reversing the sign of the latter yields the complex conjugate

$$\psi^* = a - ib$$

hence,

$$|\psi|^2 = \psi^* \psi = a^2 + b^2$$

which is positive and real.

Suppose that we are considering a beam of electrons, then it follows that

$$|\psi(x, y, z, t)|^2$$

is the average electron density at the point (x, y, z). The integral gives the average number of electrons in the volume ele-

$$\iiint |\psi|^2 dx\,dy\,dz$$

ment dxdydz. Differentiation of this integral with respect to time and setting

$$\frac{\partial}{\partial t}\iiint |\Psi|^2 dx\,dy\,dz = 0$$

verifies the conservation of electrons.

It is usually convenient to have the modulus squared equal to the probability of finding the particle described by the wave function, rather than merely accepting a proportionality. The Born normalization condition

$$\iiint |\Psi|^2 dx\,dy\,dz = 1$$

is a statement that the particle exists somewhere at all times.

Besides being normalizable, the wave function must be single-valued, and its spatial partial derivatives must be continuous everywhere.

In advanced quantum mechanics multiple wave functions are not uncommon. In Dirac[5] relativistic theory, for example, the state of a particle is described by four wave functions

$$\Psi_1,\ \Psi_2,\ \Psi_3,\ \Psi_4$$

and the modulus squared is replaced by

$$\Psi_1 \Psi_1^* + \Psi_2 \Psi_2^* + \Psi_3 \Psi_3^* + \Psi_4 \Psi_4^*$$

Relativistic quantum mechanics is used in all cases where the velocity of the particle approaches that of light.

Since most quantum variables and operators have an inherent matrix structure, it has proved convenient to express operators as eigenvalues and wave functions as eigenvectors, with mathematical operations carried out in Hilbert space.

Summarizing, matter waves represent the probability variations of finding particles in a given spatial volume at a certain time. Although at first appearing abstract, a closer examination reveals that this is in accordance with our familiar notion of a wave. The de Broglie formula enables us to calculate the wave length of the moving particle directly, whilst Schrodinger's equation yields the amplitude of the matter wave. Simple analysis reveals that the wave group associated with a moving particle travels with the same velocity as the particle; the phase velocity, however, exceeds the velocity of light.

Implications of Duality

The revelation of matter waves profoundly altered physical thought, enabling Heisenberg to formulate the uncertainty principle and Dirac to deduce the wave equation of the electron. Duality, then, spurred the development of quantum mechanics, a field which has underwritten such technological achievements as micro-electronics, lasers, atomic energy and ther-monuclear fusion. Today, quantum mechanics explores the inner sanctums of nature — deep within the atomic nucleus, seeking to unravel the perplexing puzzles of the myriad microcosm. The atomic particle appears to be the inhabitant of some unimaginable continuum, exhibiting only a phantismal existence in the macrocosmic world. Perhaps after all we shall discover that it is little more than a vortex in a microcosmic space/time structure.

Physicists optimistically anticipate the imminence of the next great step in physics; a grand panaroma with insight as its goal and boundless energy as its bonus. In a world facing energy crises, our need to understand the enigmatic heart of matter has never been greater. The abstract matter wave provides an ethereal key to the door behind which the Secrets of Nature patiently await the probing fingers of the human intellect.

Footnotes

[1] L. de Broglie, **Phil Mag,** XLVII, 446, 1924; **Ann Phys,** III, 22, 1925.

[2] Davisson and Germer, **Phys Rev,** 30, 705, 1927; **Proc Nat Acad,** 14, 317, 1928. G. P. Thomson, **Proc Roy Soc,** A, 127, 600, 1928; A, 119, 851, 1928. A. Rupp, **Ann Phys,** 85, 981, 1928.

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[4] M. Born, **Z. Phys,** XXXVII, 863, 1926.

[5] P.A.M. Dirac, **Pros Roy Soc,** A, CXVII, 610, 1928.